Introduction and Objective
Geographic tongue (GT) or benign migratory glossitis is a condition of an unknown cause characterized by chronic lesions that slowly migrate across the surface of the tongue. The condition’s characteristic wavefronts suggest that it can be modeled as a reaction–diffusion (RD) system. Here, we seek to model GT pattern evolution using RD equations.

Theory and Methods
RD systems involve, for example, chemical species reacting and diffusing across an excitable medium. Starting from the Belousov-Zhabotinsky (BZ) reaction, chemical and mathematical simplifications are made to generate nonlinear partial differential equations describing the common dynamics of all RD systems. Using the Barkley model of reaction dynamics, we have the reaction equations

\[
R_u = \frac{1}{\varepsilon} u(1 - u)(u - v + b a)
\]

\[
R_v = u - v
\]

where \(\varepsilon, a,\) and \(b\) are rate and stoichiometric parameters.

In addition to the reaction equations, we know that diffusion of any species \(w\) on a flat surface is described by

\[
\frac{\partial w}{\partial t} = D_w \nabla^2 w.
\]

To account for a curved surface, the standard Laplacian \(\nabla^2 w\) is replaced by the Laplace Beltrami Operator. Then the full reaction-diffusion equations are the sum of the reaction portion and the diffusion portion of the system:

\[
\frac{\partial u}{\partial t} = R_u + D_u \nabla^2_{LB} u \quad (1)
\]

\[
\frac{\partial v}{\partial t} = R_v + D_v \nabla^2_{LB} v \quad (2)
\]

We numerically solve (1) and (2) using the Finite Element Method and no flux boundary conditions for the back of the tongue.

The BZ reaction is the "model organism" of this research. We extract the emergent properties common to all RD systems from this reaction and apply them to GT through the Barkley model.
Results

**TABLE 1. Barkley parameters.**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon$</td>
<td>Rate constant</td>
<td>0.02</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Stoichiometric factor</td>
<td>0.75</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Stoichiometric factor</td>
<td>0.01</td>
</tr>
<tr>
<td>$D_u$</td>
<td>Activator diffusivity</td>
<td>1.0</td>
</tr>
<tr>
<td>$D_v$</td>
<td>Inhibitor diffusivity</td>
<td>0.0</td>
</tr>
</tbody>
</table>

**FIG. 6.** Geographic tongue simulation of reaction-diffusion using the finite-element method with a maximum mesh cell size of 0.1% of the total area. An elliptical pattern on an elliptic paraboloid (a) begins as a circular activator, and a spiral pattern on an oblate spheroid (b) begins as a crossed activator and an inhibitor. Blue–white–red colors code the activator $u$, and time $t$ increases downward.

**FIG. 7.** Geographic tongue simulation of reaction-diffusion on a prolate spheroid using the finite-element method, from slightly below (a) and slightly above (b). An elliptical pattern begins as a circular activator on the tongue bottom (a) and spreads to the tongue top, slowing due to the strong curvature at the tongue tip. Blue–white–red colors code the activator $u$, and time $t$ increases downward.

**FIG. 4.** Curvature effects. (a) Orthogonal osculating circles above and below a saddle point imply negative Gaussian curvature $K$. (b) Oblique view of sinusoidal surface’s positive (maxima) and negative (saddle point) curvature. (c) Top view of sinusoidal surface’s height with wavefront (white) propagating rightward. (d) Top view of corresponding curvature at a slightly later time.

**FIG. 1.** Exemplary geographic tongue on a young woman with a temporary desquamation of filiform papillae. Red dots around the wavefront are the fungiform papillae. Adapted from Martincip, Wikimedia Commons, 2012. Copyright 2012 Author(s). Licensed under a Creative Commons Attribution CC BY-SA 3.0 License.

Results

The results show that reaction–diffusion systems on curved surfaces can simulate wavefronts that are qualitatively like geographic tongue seen in patients, including ellipses and spirals. These effects can be explained by incorporating the effect of the surface curvature on the wavefront propagation speed while keeping the diffusion coefficients constant. This is different from previous work that used anisotropic diffusion to achieve oblong lesion morphology on flat tongue simulations.

Conclusions and Next Steps

Conclusion
Qualitative similarity between our simulations and patient data can be achieved without assuming anisotropic diffusion on the tongue’s surface.

Next Steps
Use experimental time-series to measure real propagation speeds and determine diffusion constants to reproduce particular subjects’ GT behavior, on the tongue or on other mucosal surfaces.

Investigate the effects on GT evolution of inert obstacles such as fungiform papillae or fissures in the tongue, which are not affected by GT, implementing anisotropic diffusion, if needed.

Lesions from other surfaces in the mouth can be modeled too.